Accelerating Innovation: National R&D Subsidies versus Foreign R&D Tax Credits

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Abstract

This paper examines and compares the impact on growth of government’s funding national R&D or providing a tax rate reduction for foreign investment in R&D. In an innovation-based model we show the relation between the costs of these two policies. One meaningful policy implication of our results is that, to accelerate innovation, governments should adopt a tax rate deduction for foreign R&D, rather than subsidizing national R&D, because the former is more economical and effective than the latter.

JEL Classification: F21; H21; O40.

Keywords: Endogenous Growth; Foreign Direct Investment; Taxes.

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1 Introduction

The emergence of a knowledge-based economy implies that economic success can only be sustained if supported by a perennial capacity to innovate and create proprietary assets. Literature in growth theory points to research and innovation as the engine of growth. Acknowledging that innovation is the key to business growth, governments start pursuing R&D strategies. European governments, under the Lisbon Strategy, are working together with funding agencies, regulatory authorities, academia and industry to create a fast-growing, dynamic research environment. The main idea is to “make Europe, by 2010, the most competitive and the most dynamic knowledge-based economy in the world” (European-Council, 2000). Areas of investigation that involve new production technologies, because of their relationship with the creation of new jobs, are especially welcomed in this context, receiving a significant part of the European Funds.

Researchers and policy-makers reckon Foreign Direct Investment (FDI) to constitute an important channel to innovation. Foreign investment increases growth through the access to better technologies. Romer (1993) emphasizes that FDI will influence the growth rate via externalities or spillover effects. Several empirical studies support this argument. (Blomström, Lipsey and Zejan, 1994, Balasubramanyam, Salisu and Sapsford, 1996, Borensztein, Gregorio and Lee, 1998, Barrel and Pain, 1997). Governments have been pursuing policies (e.g. income tax, trade policies, and subsidies to foreign firms) to attract international investments.

This paper discusses and compares the growth effect of two most common types of incentives to innovation. In practice, a country often either offer favorable tax rate to attract FDI in R&D or provide a subsidy to R&D. Our focus here is, if a government has a limited possibility of promoting growth (in terms of the resources it can use to benefit a particular economic activity) should it subsidize research activity by national firms, or should it open the economy to foreign investment and offer a tax rate reduction in the R&D sector?

Under a R&D-based growth model, comparing these two most common policies, we expect to answer unambiguously which one is more economical and efficient in terms of accelerating the rate of innovation. The results of the model suggest that while foreign investors have lower or even equal costs of introducing new goods in the economy, the government need not provide as much benefits to accelerate the innovation rate under tax credit to foreign investor as under cost subsidies to national R&D.

Even if empirical studies do not always arrive to the same conclusions for the relationship between taxes and FDI, especially because it is sometimes difficult to isolate this single effect, usually these empirical studies point to a large sensitivity of the level and location of FDI to the tax treatment it receives(Gordon and Hines, 2002). In respect to R&D incentives, Grossman and Helpman (1991, Chapter 3) demonstrate that a subsidy to this activity has positive growth results. However, the economic literature on growth is more concerned with the effects of taxes then with the specific effect of a subsidy.

In the growth literature, the ideal tax rate on income has been indicated as being asymptotically zero (Judd, 1985, Chamley, 1986). Still for closed economies but in an endogenous growth models framework, Jones, Manuelli and Rossi (1993, 1997), Bull (2002) and Milesi-Ferretti and Roubini (1995), considering the existence of human capital accumulation, also conclude that the optimal long-run tax rates is zero. Judd (2002) finds that, in the presence of imperfect

\footnote{Following a suggestion of Feenstra (1996), Reis (2001) shows for an economy with no taxes or subsidies that the welfare effect of foreign investment may be negative due to the transfer of profits to foreigners. This may dominate the positive effect of the increase in growth caused by a decrease in the cost of introducing new goods in the economy. On the other hand, Elberfeld, Götz and Stähler (2005), find that under free entry, FDI is welfare improving, since it entails a strong rationalizing effect, leading to a lower consumer price.}
competition, the ideal tax may even be negative, thus may be a subsidy. As in Reis (2006) our results also suggest a negative tax rate, since the tax credit is higher than the tax rate itself. However, taxation is particularly important in endogenous growth models where externalities can be internalized by distortionary taxation, inducing an efficient allocation (Turnovsky, 1996).

The rest of the paper is organized as follows. Section 2 lays out our R&D-based growth model, defining steady-state equilibrium. Section 3 evaluates the impact on growth of a subsidy to R&D in a closed economy framework. Section 4 evaluates the effects on growth of a tax deduction with the aim to attract FDI into the R&D sector. Section 5 compares the growth and cost effects of the two defined policies. Section 6 concludes.

2 The Model

Our R&D-based model follows Grossman and Helpman (1991, Chapter 3) and Reis (2006) but we go one step further comparing the role of public policies to accelerate the growth rate. The Government can give a subsidy to research in a closed economy or deduct taxes to profits in an open economy.

The utility of the representative household is

\[ U = \int_{0}^{\infty} e^{-\rho t} \log c_t dt, \]  

(1)

where

\[ c_t = \left( \int_{0}^{n_t} x^\alpha_j \, dj \right)^{1/\alpha}, \quad 0 < \alpha < 1. \]  

(2)

\( x_c \) represents consumption of each good \( j \) and the total set of goods in the economy is given by the interval \([0, n_t]\). Maximizing Eq. (1), we obtain the growth rate of consumption:

\[ \frac{\dot{c}}{c} = r - \rho, \]  

(3)

where \( r \) is the real interest rate. The final good is assumed to be the numeraire. Total production, \( Y_t \), takes into account how consumers’ preferences combine the different varieties. The measure of total production is

\[ Y_t = \left( \int_{0}^{n_t} x^\alpha_{P_j} \, dj \right)^{1/\alpha}, \quad 0 < \alpha < 1, \]  

(4)

where \( x_{P_j} \) is production of each good. From Eq. (4), the demand for each good is

\[ x_j = Y_t p_j^{-\varepsilon}, \]  

(5)

where \( p_j \) is the price of the intermediate good and \( \varepsilon = 1/(1 - \alpha) \) is the constant elasticity of substitution between any two varieties. A monopolist owns a patent for the production of a good \( x_j \). Labor is the only factor employed in the production, being \( L_{x_j} \) the fraction used in the production of good \( j \). The production function is then

\[ x_j = L_{x_j}. \]  

(6)

Let \( w \) be the wage rate. From the maximization of profits the price of the good \( j \) is

\[ p_j = w/\alpha \quad \text{for all } j. \]  

(7)

The wage rate is
\( w_t = \alpha Q_t, \) \hspace{1cm} (8)

and profits become:

\[ \Pi_j = (1 - \alpha) \frac{Y_t}{n_t}, \hspace{1cm} (9) \]

where \( Q_t \) is an index of knowledge in the economy, defined as \( Q_t = n_t^{(1 - \alpha)/\alpha}. \)

To develop a new good \( x_j \) the R&D sector needs a quantity \( a/n \) of labor. There are research spillovers as the cost of developing new goods depends inversely on the number of existing ones. When a researcher develops a new good owns the patent for producing it. Free-entry conditions in the R&D sector impose that the return of research for a new good must equal the cost of research. Thus,

\[ v_t = aw_t/n_t \text{ for } g > 0, \text{ with } g = \dot{n}/n, \hspace{1cm} (10) \]

where \( v_t \) is the value of a patent. There exists a tax rate over profits, \( \tau \) that is constant and known by all agents. This taxes are transferred to consumers in the form of a lump-sum transfer. With taxes \( v_t \) is given by

\[ v_t = \int_0^\infty e^{-[R(s) - R(t)]} (1 - \tau) \Pi(i) \, di, \text{ where } R(t) = \int_0^t r(i) \, di. \hspace{1cm} (11) \]

The equilibrium in the labor market is given by

\[ ag + \alpha Y_t/w_t = L \hspace{1cm} (12) \]

In this equation, the first term represents labor used in R&D whereas the second term represents labor used in the production of all varieties of goods \( x \) and is obtained from Eqs. (5)-(7).

### 3 An Optimal Subsidy in a Closed Economy

We first consider the case where the government proceeds a policy for promoting national R&D, and implements a proportional subsidy, \( s \), for the cost of research. The cost of research is now given by \( aw/n(1 + s) \) with \( 0 < s < 1 \). The new free-entry condition in the research sector is:

\[ v_t = \frac{aw}{n(1 + s)} \hspace{1cm} (13) \]

With the subsidy, the productivity of the research sector augments from \( n/a \) to \( n(1 + s)/a \).

At the steady state \( g_s \), the innovation growth rate, is constant. Then, from (12), the growth rate of production must equal the growth rate of the wage rate \( \left( \dot{Y}/Y = \dot{w}/w \right) \), which is given by \( [(1 - \alpha)/\alpha] g \). Substituting Eqs. (9) and (13) into Eq. (11), we get:

\[ \frac{Y}{w} = \frac{a[r\alpha - (1 - 2\alpha)g]}{\alpha(1 - \tau)(1 - \alpha)(1 + s)} \hspace{1cm} (14) \]

Then, substituting Eq. (14) in the labor market equilibrium condition, we get a relationship between \( g_s \) and \( r \) as:

\[ g_s = \frac{(1 - \tau)(1 - \alpha)(1 + s)L/a - r\alpha}{\alpha - (1 - \alpha)(\tau - s + s\tau)} \hspace{1cm} (15) \]

This condition is valid for \( g_s > 0 \). From the labor market condition we know that we have to verify \( g_s < L/a \) to have positive production. So, in equilibrium, \( g_s < L/a \), which implies,

\[ \frac{(1 - 2\alpha)L/a - \alpha r}{\alpha - (1 - \alpha)(\tau - s + s\tau)} < 0 \iff r > \frac{g(1 - 2\alpha)}{\alpha} \hspace{1cm} (16) \]
3.1 The Optimal Subsidy

The equilibrium interest rate equals credit supply to credit demand. Our credit supply is given by Eq. (2) and because in this economy consumption is equal to production, we have

\[ r = \frac{\dot{c}}{c} + \rho = \left( \frac{1 - \alpha}{\alpha} \right) g_s + \rho \]  

(17)

Now, using this last equation and Eq. (15) we can calculate the steady-state rate of innovation as a function of any subsidy to research:

\[ g_s = \max \left\{ \frac{(1 - \tau)(1 - \alpha)(1 + s)L/a - \alpha \rho}{1 - (1 - \alpha)(\tau - s + \tau s)} - \alpha \rho, 0 \right\} \]

(18)

Figure 1 provides a sensitivity analysis of the growth rate value, Eq. (18), with respect to the parameters of the model: \( \varepsilon \) and \( s \). The simulation is performed against the Portuguese case (see Appendix for description of the data). It is revealed that \( g_s \) rises when \( s \) is high and \( \varepsilon \) moves towards one. Figure 1 also shows that \( g_s \) is much more sensitive to \( \varepsilon \) than to \( s \). This is due to the fact that with a more elastic demand\(^2\), a higher price dampens more the increase in profits, implying that the net increase in profits falls short of the entry of new firms in the R&D sector. Similarly, with a less elastic demand (\( \varepsilon \approx 1 \)), a higher price dampens less the increase in profits, leading to more firms’ entry into the R&D sector.

\( \text{(Insert Figure 1 here)} \)

**Proposition 1** For a closed economy, the optimal subsidy increases with the tax rate, \( \tau \), and with the labor force, \( L \), and decreases with the innovation cost, \( a \), with the intertemporal discount factor, \( \rho \), and with \( \alpha \), \( s = \left[ \frac{L/a\rho + \tau - \alpha/ (1 - \alpha) - \alpha \rho}{1 - (1 - \alpha)(\tau - s + \tau s)} \right] \)

\( \)\( \rho \)

**Proof.** To determine the optimal subsidy, we have to calculate consumption in the closed economy using Eqs. (12) and (18):

\[ c = \frac{w(L + a \rho)}{[1 - (1 - \alpha)(\tau - s + \tau s)]} \]

(19)

Now we use Eqs. (1), (8), (18) and (19) to obtain the utility function as

\[ U = \frac{1}{\rho \alpha}[ \log \left( \frac{L + a \rho}{1 - (1 - \alpha)(\tau - s + \tau s)} \right) + \log \alpha Q_0] 
+ \frac{1}{\rho^2} \left( \frac{1 - \alpha}{\alpha} \right) \frac{(1 - \tau)(1 - \alpha)(1 + s)L/a - \alpha \rho}{1 - (1 - \alpha)(\tau - s + \tau s)} \]

(20)

and calculate the effect of the subsidy on welfare

\[ \frac{dU}{ds} = \frac{(1 - \tau)(1 - \alpha)}{\rho^2 [1 - (1 - \alpha)(\tau - s + \tau s)]^2} [(1 - \alpha)L/a + \rho (1 - \alpha)(\tau - s + \tau s) - \alpha \rho] \]

(21)

Thus, \( s \) is given by

\[ s = \frac{1}{1 - \tau} \left[ \frac{L}{a \rho} + \tau - \frac{\alpha}{1 - \alpha} \right] \]

(22)

\(^2\)Since a monopolist competitor operates on the part of the demand curve where demand is less elastic, it follows that it is reasonable to assume the elasticity of demand to be around two.
The subsidy, $s$, as given by Eq. (22) is optimal only if it implies $g_s > 0$. Substituting Eq. (22) in Eq. (18) we obtain

$$g(s) = \frac{L(1-\alpha) - a\alpha p}{a(1-\alpha)}$$  \hspace{1cm} (23)

The growth rate is positive if $L/a > \alpha p/(1-\alpha)$, which implies that $s > 0$. This result is in accordance with Grossman and Helpman (1991) because it implies that in the presence of externalities from R&D, the equilibrium growth rate will be higher with a policy that can internalize such spillovers. A subsidy may improve the private benefits of R&D motivating firms to enhance research.

4 Tax Deduction in an Open Economy

Consider a small economy, open to foreign investment and where foreign investors borrow in the international capital market. The interest rate is given by the international interest rate, $r = r^*$ and the supply of credit is infinitely elastic. The cost of innovation may be identical between national and foreign investors or may be smaller for foreign investors because they are introducing in our economy a set of new goods that have been developed and produced elsewhere. In any case, there will exist an unique production cost that will be equal to the international one. If $a^* < a$ all innovation will end up being done by foreign investors. Assume government gives a tax reduction, $d$, to foreigners in order to stimulate FDI and thus the innovation rate. The value of the patent, $v_t$, is now given by:

$$v_t = \int_{t}^{\infty} e^{-[R(s)-R(t)]} (1-\tau+d) \Pi(t) \, dt,$$

where

$$R(t) = \int_{0}^{t} r(\iota) \, d\iota.$$  \hspace{1cm} (24)

At the steady state the innovation rate, $g_d$, is again constant. The value of a patent becomes:

$$v_t = \frac{(1-\tau+d)(1-\alpha)Y_t}{r - \frac{1-2\alpha}{\alpha}g}$$  \hspace{1cm} (25)

The new labor market equilibrium condition for the open economy is given by:

$$a^*g_d + \alpha Y_t / w_t = L$$  \hspace{1cm} (26)

Substituting Eq. (25) and the free-entry condition in Eq. (26) we obtain the innovation growth rate as a function of the international interest rate

$$g_d = \frac{(1-\tau+d)(1-\alpha)L/a^* - \alpha r^*}{\alpha - (\tau-d)(1-\alpha)}.$$  \hspace{1cm} (27)

Once again, Eq. (27) is valid for $g_d > 0$. From the labor market condition we know that $g_d < L/a^*$. So, in equilibrium we must verify that

$$\frac{(1-2\alpha)L/a^* - \alpha r}{\alpha - (\tau-d)(1-\alpha)} < 0$$  \hspace{1cm} (28)

3If international firms have the same production cost they may produce in the internal market and the innovation cost remains unchanged. In equilibrium, the set of goods produced in this economy will be produced by nationals and foreigners. If foreign investors have a smaller innovation cost they will replace the national firms and the cost of innovation will be determined by the foreign firms. In equilibrium all production will be done by foreign firms.
After opening to FDI, the credit supply function becomes

$$ r^* = \frac{\dot{c}}{c} + \rho = \left( \frac{1 - \alpha}{\alpha} \right) g + \rho $$

(29)

The steady state rate of innovation after the tax deduction, using Eqs. (27) and (29) is:

$$ g_d = \max \left\{ \frac{(1 - \tau + d)(1 - \alpha) L/a^* - \alpha \rho}{1 - (\tau - d)(1 - \alpha)}, 0 \right\} $$

(30)

The sensitivity analysis of the growth rate value, Eq. (30), with respect to the parameters of the model: $\varepsilon$ and $d$, is illustrated in Figure 2. The simulation reveals that $g_d$ rises when $d$ is high and $\varepsilon$ moves towards one. Comparing Fig. 1 with Fig. 2, the innovation rate is higher when the economy opens to FDI.

(Insert Figure 2 here)

**Proposition 2** Consider an economy open to foreign investment. Assume that

$$ r^* = \left( \frac{1 - \alpha}{\alpha} \right) \frac{L}{a^*} $$

The optimal tax credit has to verify $d - \tau > 0$, which means that it is equivalent to a subsidy.

**Proof.** From Eqs. (26) and (30) consumption in the open economy is given by

$$ c = \frac{w (L + a^* \rho)}{1 - (\tau - d)(1 - \alpha)} $$

(31)

Use Eqs. (1), (8), (30) and (31) to obtain the utility function:

$$ U = \frac{1}{\rho} \left[ \log \frac{L + a^* \rho}{1 - (\tau - d)(1 - \alpha)} + \log aQ_0 \right] + \frac{1}{\rho^2} \left( \frac{1 - \alpha}{\alpha} \right) \frac{(1 - \tau + d)(1 - \alpha) L/a^* - \alpha \rho}{1 - (\tau - d)(1 - \alpha)} $$

(32)

and derive the optimal tax deduction:

$$ \frac{dU}{dd} = 0 \iff d - \tau = \frac{L}{a^* \rho} - \frac{\alpha}{1 - \alpha} $$

(33)

Substituting this result in Eq. (30), we obtain

$$ g(d) = \frac{(1 - \alpha) L - \alpha a^* \rho}{a^* (1 - \alpha)} $$

(34)

The growth rate is positive if $L/a^* > \alpha \rho / (1 - \alpha)$, which implies that $d - \tau > 0$. ■

This result shows that to stimulate the rate of innovation, the government would have to give a subsidy to foreign investors in order to attract them into R&D sector. This is again the Grossman and Helpman (1991) result that states that government has to internalize research spillovers to augment growth.

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4Since we are assuming that all the production is done by foreign investors we still verify that consumption is equal to production.

5The parameters are calibrated with the values shown in Table 1. of the Appendix. Here we use United States’ innovation cost.
5 Subsidy or Tax Credit: Which One Is the Most Efficient?

In this section, first, we compare the cost of the tax rate deduction and the subsidy, while holding the innovation costs of domestic and foreign firms equal. We then show that the steady state rate of innovation under the FDI’s tax rate reduction is higher than under subsidy to R&D, when foreigners investors have a lower innovation cost.

Proposition 3 Consider a closed economy and an economy open to foreign investment with the same innovation cost. Then, the value of the tax rate deduction is smaller than the subsidy to R&D.

Proof. From Eqs. (23) and (34), the steady state growth rates for the subsidy and for the tax deduction are given by:

\[
g(s) = \frac{(1-\alpha)L-a\alpha\rho}{a(1-\alpha)} \quad g(d) = \frac{(1-\alpha)L-a^*\alpha\rho}{a^*(1-\alpha)}
\]

respectively. These growth rates differ only from the productivity of the research sector \(1/a\) or \(1/a^*\). If the productivity in research of foreign investors equals the productivity in research by national firms\(^6\) our results show that the economy would have the same steady state growth rate given by (23).

To answer which of the policies is more economical and efficient in terms of accelerating the rate of innovation, we compare their values. Recalling Eqs. (22) and (33) for the values of the subsidy and of the tax deduction, respectively, we have

\[
s = \frac{1}{1-\tau} \left[ \frac{L}{a\rho} + \tau - \frac{\alpha}{1-\alpha} \right] \quad d = \frac{L}{a^*\rho} + \tau - \frac{\alpha}{1-\alpha}.
\]

Given that \(1/(1-\tau) > 1\), it is then straightforward to conclude that the subsidy is a more expensive policy to achieve the same growth results than tax rate deduction. Governments should adopt a tax rate deduction for foreign R&D, rather than subsidizing national R&D, because the former is more economical and effective than the latter. ■

Proposition 4 Consider that foreign investors have a lower innovation cost, then the home country would grow faster opening to FDI and offering a tax credit to foreign investors.

Proof. Comparing \(g(s)\) and \(g(d)\) as given by Eqs. (23) and (34), respectively, and noting that \(a^* < a\), it is straightforward to conclude that with the tax rate deduction, after opening the economy to foreign investors, the economy grows faster: \(g(d) > g(s)\). ■

Thus, if multinational enterprizes have a lower innovation cost, the economy steady state growth rate is higher when opened to FDI than when closed, since the growth rate depends on the innovation cost. Then, we have growth effects, which can be seen by the expression of the steady state growth rate.

(Figure 3 illustrates the relationship between Eqs. (23) and (34))\(^7\). It shows that when the innovation cost is high (closed economy case), the rate of innovation is low; whereas when the innovation cost is low (open economy case) the innovation rate is high.

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\(^6\)This could be explained by the globalization of activities, conducting the economies to the same activity costs.

\(^7\)The parameters are calibrated with the values shown in Table 1. of the Appendix.
6 Conclusion

The growth literature points to the importance of research and development as the engine of economic growth. Spillover effects of technology activities to the private ones are the main responsible for this positive growth effects, so economic policies should promote and internalize them. These technology developments can occur at the national level in private research laboratories that can have government subsidization or can be imported directly from abroad through international investments by multinational enterprises. Sometimes to attract these foreign firms governments have to pursue competitive fiscal policies like tax deductions.

In this paper, we compare two policies intended to stimulate the rate of innovation: one that subsidizes national R&D activity and a policy that offers a tax deduction to attract FDI in R&D. First, we compare a situation in which the foreign firms enter a small open economy with the same innovation costs as the national firms in the R&D sector. In this case, deducting taxes to foreign firms appears to be more efficient and economical. Second, we consider the hypothesis of international firms having a smaller innovation cost. In this situation our results are reinforced by the fact that the national steady state growth rate becomes higher.
References


Appendix

Simulations

The simulations relate to the growth rates obtained in Eqs. (18), (30), (23) and (34). These simulations are conducted with reference to the Portuguese case. The values of the parameters, as well as the ranges used in the simulations of growth rates, were drawn the European Commission, Economic and Financial Affairs, (AMECO Database) for the period 1998–2002 and from UIS Science and Technology database for the year 2000. Thus, the parameters from the equations of the growth rate are defined as:

\( \tau \): Tax rate is the Portuguese “current tax on income and wealth: corporations”, from AMECO database, for the period 1998–2002.

\( \alpha \): The parameter of the elasticity of substitution between any two products, \( \varepsilon \), being \( \varepsilon = 1/(1-\alpha) > 1, \ 0 < \alpha < 1 \).

\( \rho \): Discount rate is the United States “real long-term (5 years) interest rate, deflator GDP”, from AMECO database, for the period 1998–2002.

\( L \): Labor force is the Portuguese “Total labor force”, from AMECO database, for the period 1998–2002.

\( a \): Total gross domestic expenditure on R&D (GERD) \( \times 1000 \) / Full-time equivalent researchers \( \times \) GERD as % GDP, Portugal, from (UIS) database, for the year 2000.

\( a^* \): Total gross domestic expenditure on R&D (GERD) \( \times 1000 \) / Full-time equivalent researchers \( \times \) GERD as % GDP, USA, from (UIS) database, for the year 2000.

\( s \): Since the data on subsidies to innovation costs where not available, the value and the range of variation were picked arbitrarily.

\( d \): Since the data on tax deduction to profits where not available, the value and the range of variation were picked arbitrarily.

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**Figure 1:** $\tau = 0.16$, $\mu = 0.04$, $a = 68019$, $L = 5230.5$

**Figure 2:** $\tau = 0.16$, $\mu = 0.04$, $a^* = 56108$, $L = 5230.5$
Figure 3: $\mu = 0.04, L = 5230.5$